## Problem 1.A. (Solution):

1.A.1: At the end of the motion the ball will be at rest. It means that the angular momentum of the ball will be zero. Let us consider the angular momentum of the ball referred to point $P$. In this way the torque of the kinetic frictional force is always zero referred to this point, so the angular momentum is constant and this constant is zero. This means that the impulse of the force $\boldsymbol{F}$ causes a zero angular momentum referred to point $P$. The only way to do this if the line of action of force $\boldsymbol{F}$ is going through point $P$ :


It can be seen on the figure that $c=2 R \sin \alpha$ and $\sin \alpha=\frac{h}{c}$. It means that

$$
\sin \alpha=\sqrt{\frac{h}{2 R}}<1
$$

1.A.2: The angular velocity of the ball has three components $\left(\omega_{x} ; \omega_{y} ; \omega_{z}\right)$ while the velocity of the ball has only two components $\left(v_{\mathrm{x}} ; v_{\mathrm{y}}\right)$ as it can be seen in the figure:


Let us investigate the velocity components of point $P$ :

$$
\begin{aligned}
& v_{P \mathrm{x}}=v_{\mathrm{x}}-R \omega_{\mathrm{y}} \\
& v_{P \mathrm{y}}=v_{\mathrm{y}}+R \omega_{\mathrm{x}}
\end{aligned}
$$

We can take the derivatives of all the terms of these equations:

$$
\begin{gathered}
\dot{v}_{P \mathrm{x}}=\dot{v}_{\mathrm{x}}-R \dot{\omega}_{\mathrm{y}} \\
\dot{v}_{P \mathrm{y}}=\dot{v}_{\mathrm{y}}+R \dot{\omega}_{\mathrm{x}}
\end{gathered}
$$

The acceleration and angular acceleration of the ball is caused by the kinetic frictional force:

$$
\dot{v}_{\mathrm{x}}=\frac{S_{\mathrm{x}}}{m} ; \quad \dot{v}_{\mathrm{y}}=\frac{S_{\mathrm{y}}}{m} ; \quad \dot{\omega}_{\mathrm{x}}=\frac{S_{\mathrm{y}} R}{\frac{2}{5} m R^{2}} ; \quad \dot{\omega}_{\mathrm{y}}=\frac{-S_{\mathrm{x}} R}{\frac{2}{5} m R^{2}} .
$$

Inserting these results into the equations above, it yields

$$
\dot{v}_{P \mathrm{x}}=\frac{7}{2 m} S_{\mathrm{x}} \quad \text { and } \quad \dot{v}_{P \mathrm{y}}=\frac{7}{2 m} S_{\mathrm{y}} \quad \rightarrow \quad \dot{\vec{v}}_{P}=\frac{7}{2 m} \vec{S}=-\frac{7}{2} \mu g \frac{\vec{v}_{P}}{\left|\vec{v}_{P}\right|} .
$$

It means that the direction of velocity of point $P$ is constant and it decreases uniformly to zero with an acceleration of $-\frac{7}{2} \mu g$. You can see that not only the magnitude but the direction of the kinetic frictional force is also constant. This direction is different than the direction of CM of the ball, so the ball is moving along a parabolic trajectory. When the velocity of point $P$ becomes zero, the frictional force vanishes and the ball will do uniform rectilinear motion (which is tangential to the parabola).

